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Dispersion of meteor trails in the geomagnetic field

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A meteor trail is modeled by a long column of weakly ionized plasma, whose dispersion is controlled by the geomagnetic field and the requirement to maintain effective space charge neutrality. First we consider scattering of a radar signal from an underdense trail and derive an expression for the amplitude of the backscattered signal as a function of time. Then, starting from the basic momentum balance equations for electrons and ions in a partially ionized plasma, we require divergences of ion and electron fluxes to be equal, plus assume equality of the flux components along the magnetic field direction. The analysis is really applicable to a whole range of plasma problems, although we focus upon meteor trails for now. It is found that charged particle densities satisfy a diffusion equation and we obtain an expression for the ambipolar diffusion tensor and expressions for the ambipolar electric field, valid for arbitrary relative orientations of the magnetic field and meteor trail axis. Results are somewhat different from previous analyses in the meteor literature.

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I. INTRODUCTION

The theoretical description of the dispersion of meteor trails at heights $\gtrsim 95$ km, where atmospheric density is sufficiently low such that collision frequencies are small enough to allow the geomagnetic field to significantly influence transport properites, has been the subject of a number of studies over the years. The paper by Jones [1] outlines the traditional approach to the problem and summarizes the earlier literature [2,3]. More recently, the Elford brothers [4] have discussed the numerical calculation of the effective diffusion coefficients appearing in Jones' paper using ''swarm'' and atomic physics data.

The ionized gas in the trail generated by the passage of a meteor in the upper atmosphere generally exhibits the properties of a low temperature, quiescent plasma. In particular, the subsequent diffusion is ambipolar. An understanding of ambipolar diffusion in plasmas subject to a magnetic field is of importance in a wide variety of circumstances, from both laboratory low and high temperature plasmas to naturally occurring phenomena. Unfortunately, the theory seems to be problematic, judging from the remarks of Phelps [5], often contradictory text book presentations [6,7], and an apparently completely different way of looking at things in the meteor literature [1–3]. Little attempt has hitherto been made to place the meteor problem in the context of mainstream plasma physics. A comparison is long overdue and is the first and primary task of the present paper. It is by no means a trivial operation. The difficulty is compounded by the fact that ambipolar diffusion in a magnetic field *per se* also needs to be examined. Thus although we discuss the problem in the context of dispersion of meteor trails, the results have a wider applicability, at least for cylindrical plasmas in a uniform field. Because of the dual purpose nature of this paper, we have chosen to limit the discussion more towards generalities, preferring to leave detailed applications and numerical calculations (to meteor trials or otherwise) to subsequent specialized papers. This then is the scope of the present paper.

We commence with a brief discussion in Sec. II of a radar signal backscattered from the electrons in an ''underdense'' meteor trail. Irrespective of the initial shape of the trail, the intensity of the signal decays exponentially, but the time constant is determined by an effective diffusion coefficient which is quite different from anything which appears in the meteor literature. Indeed the results obtained here are often at odds with this previous literature, except in certain limiting cases.

The transport theory providing the basis for our expressions is given in Sec. III, while the results are discussed in Sec. IV.

II. THE BACKSCATTERED SIGNAL

Consider a meteor trail to be a column of partially ionized plasma oriented at an arbitrary angle θ to the geomagnetic field $\mathbf{B}$. A radar transmitter at the earth's surface sends a signal of wave number $\mathbf{k}_0$ to a region of the magnetosphere, where it is scattered by the column into an outgoing signal of wave number $\mathbf{k}$. The coordinate system is shown in Fig. 1.

The amplitude $A(t)$ of the scattered wave is proportional to the Fourier transform of the electron number density $n(\mathbf{r},t)$, i.e.,

$$A(t) \sim \hat{n}(\Delta\mathbf{k},t) = \int d^3\mathbf{r}\, n(\mathbf{r},t)\exp(-i\Delta\mathbf{k}\cdot\mathbf{r}), \qquad (1)$$

where $\Delta\mathbf{k} = \mathbf{k} - \mathbf{k}_0$ is the change in wave vector. For backscattering, $\mathbf{k} = -\mathbf{k}_0$, and it is clear that

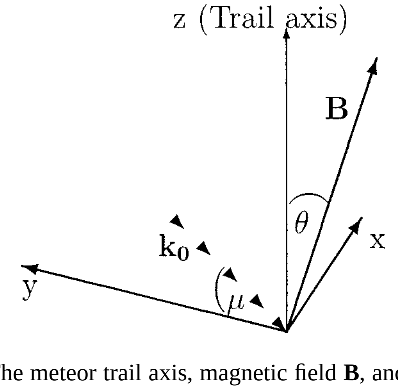

FIG. 1. The meteor trail axis, magnetic field $\mathbf{B}$, and wave vector $\mathbf{k}_0$.

$$\Delta \mathbf{k} = -2\mathbf{k}_0 . \tag{2}$$

In the next section we show that electron density satisfies a diffusion equation, which can be written in the form

$$\partial_t n = \mathsf{D}:\nabla\nabla n \tag{3}$$

in which the colon denotes a double contraction over tensor indices, the diffusion tensor has the structure

$$\mathsf{D} = D_\parallel \mathbf{bb} + D_\perp (\mathsf{I} - \mathbf{bb}), \tag{4}$$

where $\mathbf{b}$ is a unit vector in the direction of $\mathbf{B}$, I is the unit tensor, and $D_\parallel$and $D_\perp$ are diffusion coefficients parallel and perpendicular to $\mathbf{B}$. To find $\hat{n}(\mathbf{K},t)$ we first Fourier transform Eq. (3) and then solve the resulting equation:

$$\hat{n}(\mathbf{K},t) = \hat{n}(\mathbf{K},0)\exp\{-\mathbf{KK}:\mathsf{D}t\}. \tag{5}$$

With $\mathbf{K} = \Delta\mathbf{k} = -2\mathbf{k}_0$ it is clear from Eqs. (1) and (5) that the scattered amplitude is given by

$$A(t) = A(0)\exp\{-4\mathbf{k}_0\mathbf{k}_0:\mathsf{D}t\}. \tag{6}$$

This expression is valid whatever the initial distribution $n(\mathbf{r},0)$, whose effect is manifest in the backscattered signal through the constant amplitude $A(0) \sim \hat{n}(-2\mathbf{k}_0,0)$.

We now consider the coordinate system of Fig. 1, in which the z axis defines the direction of the meteor trail, the x-z plane contains the geomagnetic field, while the incident/backscattered waves are contained in the x-y plane. Thus the angle between $\mathbf{B}$ and the z axis is θ, while we denote by μ the angle between $\mathbf{k}_0$ and the y axis. In this reference frame,

$$\mathbf{b} = \sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{k}, \tag{7}$$

$$\mathbf{k}_0 = -k_0 \sin\mu\,\mathbf{i} - k_0 \cos\mu\,\mathbf{j}, \tag{8}$$

and hence

$$\mathbf{k}_0\cdot\mathbf{b} = -k_0 \sin\mu \sin\theta, \tag{9}$$

$$\mathbf{k}_0\mathbf{k}_0:\mathsf{D} = k_0^2[D_\parallel \sin^2\mu \sin^2\theta + D_\perp(1-\sin^2\mu \sin^2\theta)]. \tag{10}$$

When Eq. (10) is substituted in Eq. (6) there results

$$A(t) = A(0)\exp[-4k_0^2 t D_{\mathrm{eff}}], \tag{11}$$

where the effective orientation-dependent diffusion coefficient is given by

$$D_{\mathrm{eff}} = D_\parallel \sin^2\mu \sin^2\theta + D_\perp(1-\sin^2\mu \sin^2\theta)]. \tag{12}$$

We now turn to the transport theory which produces these expressions.

III. AMBIPOLAR DIFFUSION IN A PLASMA COLUMN

A. Momentum balance equations and key assumptions

The equations of continuity for electron and ions are, assuming no bulk ionization or attachment processes,

$$\partial_t n_e + \nabla\cdot\Gamma_e = 0, \tag{13}$$

$$\partial_t n_i + \nabla\cdot\Gamma_i = 0, \tag{14}$$

where Γ_e and Γ_i denote electron and ion particle fluxes, respectively. We shall assume singly charged ions and that the plasma column has evolved to a state where quasineutrality has been attained, so that

$$n_e \approx n_i \equiv n. \tag{15}$$

It must be emphasised that this is an approximation, there being some charge separation and some deviation from strict neutrality, resulting in an ambipolar electric field $\mathbf{E}$ being set up. The accuracy of the approximation and the internal consistency of the treatment is something that is often taken for granted. In any case, in what follows we impose the condition

$$\nabla\cdot\Gamma_e = \nabla\cdot\Gamma_i \equiv \nabla\cdot\Gamma, \tag{16}$$

which follows from Eqs. (13) and (14) and the quasineutrality condition (15). Notice that Eqs. (15) and (16) are consistent with the assumptions of Ref. [1]. Similar assumptions have been made in a recent discussion on ambibolar ''diffusion cooling'' [8].

The momentum balance equations for ions and electrons in a neutral gas are [9]:

$$kT_e\nabla n + e(n\mathbf{E} + \mathbf{\Gamma}_e\times\mathbf{B}) = -m_e\nu_e\mathbf{\Gamma}_e, \tag{17}$$

$$kT_i\nabla n - e(n\mathbf{E} + \mathbf{\Gamma}_i\times\mathbf{B}) = -\mu_i\nu_i\mathbf{\Gamma}_i, \tag{18}$$

where Γ_e and Γ_i denote electron and ion particle fluxes, ν_e and ν_i are collision frequencies for momentum transfer between electrons and neutral molecules and ions and neutrals, respectively, m_e is the electron mass, and μ_i is the reduced mass of an ion-neutral pair. Electron and ion temperatures are represented by T_e and T_i, respectively. Equations (17) and (18) convert to the familar flux-gradient relationships [1–3,9] upon solving for Γ_e and Γ_i, but we find it more convenient to leave them in their original form. It is to be emphasized that we are explicitly assuming that the ambipolar field E is sufficiently weak so that collision frequencies and temperatures are all constants, independent of E. Otherwise, if E were strong, a very difficult nonlinear problem would result. This assumption is implicit in all other treatments of ambipolar diffusion.

In the absence of a magnetic field, it is usually assumed that all components of the electron and ion particle fluxes are equal [6,7]. In the more general case, which is presently being discussed, it is possible to equate the components of particle flux along $\mathbf{B}$ only, for only in that direction is charged particle motion unaffected by $\mathbf{B}$. Thus we have

$$\mathbf{b}\cdot\Gamma_e = \mathbf{b}\cdot\Gamma_i \equiv \Gamma_\parallel . \tag{19}$$

Otherwise, only equality (16) of the divergences of particle fluxes can be assumed.

Note that in the above and in what follows we denote by subscripts $\parallel$ and $\perp$ properties and operations parallel and perpendicular to $\mathbf{B}$, respectively. (Note that the direction of the axis of the meteor trail plays no special role when it comes to calculation of transport properties.) Thus, for example,

$$E_{\parallel}=\mathbf{b}\cdot\mathbf{E}, \tag{20}$$

$$\mathbf{E}_{\perp}=(\mathsf{I}-\mathbf{bb})\cdot\mathbf{E}. \tag{21}$$

Finally, we note two further important points:

(i) It is either explicitly or implicitly assumed here and in other work that the magnetic field is constant in both space and time. This is clearly an approximation, but if we accept it, then by Faraday's law it is required that $\mathbf{b}\cdot\mathrm{curl}\,\mathbf{E}=\mathbf{0}$ for consistency.

(ii) In the simplest case where the magnetic field is directed along the axis of the column, symmetry arguments [6] lead to the conclusion that the ambipolar electric field $\mathbf{E}$ has no azimuthal component perpendicular to both $\mathbf{B}$ and ∇n, a condition expressed formally by $\mathbf{E}\cdot(\nabla n\times\mathbf{B})=0$. Note that there may be currents circulating in the $\nabla n\times\mathbf{B}$ direction, but these do not result in any charge separation, and therefore do not produce an ambipolar field. We argue that this condition also prevails in the more general case where $\mathbf{B}$ makes an arbitrary angle with the plasma column.

Putting these last two requirements together, we have

$$\mathbf{b}\cdot\mathrm{curl}(n\mathbf{E})=n\mathbf{b}\cdot\mathrm{curl}\,\mathbf{E}+\nabla n\cdot(\mathbf{E}\times\mathbf{b})=0. \tag{22}$$

In the meteor literature [1–3] the trail axis rather than the magnetic field is considered to provide the controlling reference direction. Thus it is usually assumed that $\mathbf{E}$ derives from a scalar potential and any component along the trail axis is implicitly suppressed. This coincides with our scenario only in the special case where $\theta=0$.

B. Parallel diffusion

The components of the balance equations along the magnetic field are found by taking the dot product of Eqs. (17) and (18) with the unit vector $\mathbf{b}$:

$$kT_e\nabla_{\parallel}n+neE_{\parallel}=-m_e\nu_e\Gamma_{\parallel}, \tag{23}$$

$$kT_i\nabla_{\parallel}n-neE_{\parallel}=-\mu_i\nu_i\Gamma_{\parallel}. \tag{24}$$

Elimination of $E_{\parallel}$ gives

$$\Gamma_{\parallel}=-D_{\parallel}\nabla_{\parallel}n, \tag{25}$$

where

$$D_{\parallel}=\frac{kT_e+kT_i}{m_e\nu_e+\mu_i\nu_i}\approx\frac{kT_e+kT_i}{\mu_i\nu_i}\equiv D_i\left(1+\frac{T_e}{T_i}\right) \tag{26}$$

and $D_i=kT_i/\mu_i\nu_i$ is the free ion diffusion coefficient. Similarly, elimination of $\Gamma_{\parallel}$ gives

$$neE_{\parallel}\approx-kT_e\nabla_{\parallel}n. \tag{27}$$

All these equations are the same as the textbook formulas for $B=0$, which is to be expected, as motion along $\mathbf{B}$ is the same as if no field were present. Note that nonzero $E_{\parallel}$ implies the existence of a nonzero component of $\mathbf{E}$ along the trail axis, again at odds with [1–3]

C. Transverse diffusion, diffusion equation

If the divergence is taken of Eqs. (17) and (18) there result equations containing not only $\nabla\cdot\Gamma$ but also curl Γ. Further vector operations and manipulations eventually lead to the expressions

$$\nabla_{\perp}\cdot\Gamma_{\perp}=D_{\perp}\nabla_{\perp}^2 n, \tag{28}$$

$$\nabla_{\perp}\cdot ne\mathbf{E}_{\perp}=-\frac{(kT_e-\rho kT_{i)}}{1+\rho}\nabla_{\perp}^2 n, \tag{29}$$

where

$$D_{\perp}=\frac{kT_e+kT_i}{\mu_i\nu_i(1+\rho)}=\frac{D_{\parallel}}{1+\rho} \tag{30}$$

is the transverse ambipolar diffusion coefficient, and

$$\rho\equiv\frac{e^2B^2}{m_e\nu_e\mu_i\nu_i} \tag{31}$$

is the product of the cyclotron frequency-collision frequency ratios for ions and electrons.

Equations (25) and (28) combine to furnish the total divergence of the particle flux

$$\nabla\cdot\Gamma=-D_{\parallel}\nabla_{\parallel}^2 n-D_{\perp}\nabla_{\perp}^2 n \tag{32}$$

and this together with the equation of continuity [either Eq. (13) or Eq. (14)] gives the diffusion equation

$$\partial_t n=D_{\parallel}\nabla_{\parallel}^2 n+D_{\perp}\nabla_{\perp}^2 n\equiv\mathsf{D}:\nabla\nabla n, \tag{33}$$

where the diffusion tensor D is expressed by Eq. (4) above.

Note that the transverse diffusion coefficient [Eq. (30)] is independent of the angle of orientation of the column with respect to $\mathbf{B}$, as it should be. Transport properties are controlled by collision frequencies, densities, temperature, external fields, etc, but not in general by geometry. (An exception to this is associated with the ''diffusion cooling'' phenomenon [8].)

D. The ambipolar electric field

The diffusion equation (33) and the analysis in Sec. III C effectively solves the problem (at least formally) without any further ado. However, it is of interest to discuss the ambipolar electric field further. We have an expression (27) for the component of $\mathbf{E}$ parallel to $\mathbf{B}$, and for the transverse component we assume that Eq. (29) implies

$$ne\mathbf{E}_\perp = -\frac{(kT_e - \rho kT_i)}{1+\rho}\nabla_\perp n. \tag{34}$$

Equations (27) and (35) can be combined to give the complete expression for the ambipolar field:

$$ne\mathbf{E} = -kT_e[\Delta\mathsf{I} + (\mathsf{I}-\Delta)\mathbf{bb}]\cdot\nabla n \tag{35}$$

with

$$\Delta = \frac{1-\rho(T_i/T_e)}{1+\rho}. \tag{36}$$

Let us take the same coordinate system as before, in which the z axis is defined by the axis of the plasma column, and $\mathbf{B}$ lies in the x-z plane, making an angle θ with the z axis. Then the components of $\mathbf{E}$ are

$$neE_x = -kT_e[\Delta\partial_x n + (1-\Delta)\sin\theta(\sin\theta\partial_x n + \cos\theta\partial_z n)], \tag{37}$$

$$neE_y = -\Delta kT_e\partial_y n, \tag{38}$$

and

$$neE_z = -kT_e[\Delta\partial_z n + (1-\Delta)\cos\theta(\sin\theta\partial_x n + \cos\theta\partial_z n)]. \tag{39}$$

If the column is uniform along its axis, then $\partial_z n = 0$ and Eqs. (37) and (39) simplify to

$$neE_x = -kT_e[\Delta + (1-\Delta)\sin^2\theta]\partial_x n, \tag{40}$$

$$neE_z = -kT_e(1-\Delta)\sin\theta\cos\theta\partial_x n. \tag{41}$$

Suppose that B and hence ρ are sufficiently large that Δ defined by Eq. (36) becomes negative. Then by Eq. (40) E_x can be zero or negative (i.e., $\mathbf{E}$ has a component pointing radially inwards), depending upon whether $\theta \leqslant \theta_c$, where

$$\sin^2\theta_c \equiv \frac{|\Delta|}{1+|\Delta|}. \tag{42}$$

That a radial ambipolar field can change in sign has long been known [10]. Finally, notice that by Eqs. (37) and (39),

$$\mathbf{b}\cdot\mathrm{curl}(n\mathbf{E}) = \sin\theta\{\mathrm{curl}(n\mathbf{E})\}_x + \cos\theta\{\mathrm{curl}(n\mathbf{E})\}_z \equiv 0 \tag{43}$$

confirming Eq. (22) and the internal consistency of the calculation.

E. Use of swarm data

To be of practical use, the formulas above must be expressible in terms of empirically measured quantities. In swarm experiments [11] free diffusion coefficients D and mobilities K of electrons and ions in gases are measured over a range of applied electric fields. For weak electric fields, the relationships of the latter to the collision frequencies appearing in this work are

$$K_e \equiv \frac{e}{m_e\nu_e}, \quad K_i \equiv \frac{e}{\mu_i\nu_i} \tag{44}$$

and hence, for example, the key parameter [Eq. (31)] is expressible as

$$\rho = K_e K_i B^2. \tag{45}$$

Electron and ion mobilities are tabled extensively in the literature (see, e.g., references cited by Elford and Elford [4] and [11]). The free ion diffusion coefficient also figures prominently in the discussion. If it is not directly available from swarm experiments, then it may be deduced from $D_i = (kT_i/e)K_i$, effectively the Einstein relation [11]. Note that estimates of electron and ion temperatures are required to complete any numerical calculations. In any case, the expressions for ambipolar diffusion coefficients can be evaluated numerically from existing empirical swarm data, for whatever type of plasma is desired, for meteor trails in the geomagnetic field or otherwise.

IV. CONCLUDING REMARKS

We have analyzed ambipolar diffusion in a cylindrical column of partially ionized plasma oriented at an arbitrary angle with respect to a uniform magnetic field, with a view to applying the results to dispersion of meteor trails in the ionosphere, and to clarifying uncertainties from a fundamental plasma physics point of view [5–7]. Our approach is consistent with conventional plasma physics ideas, but our results appear to be at odds with the meteor literature [1–3]. We have started with the basic momentum balance equations for ions and electrons in a weakly ionized plasma and derived expressions (26) and (30) for ambipolar diffusion coefficients parallel and perpendicular to the magnetic field. It has also been shown that the radial component of the ambipolar electric field can reverse sign for sufficiently small relative orientation angles.

Present thinking differs fundamentally from other papers in the meteor literature, insofar as the magnetic field direction provides the reference axis for the calculation of transport properties, not the axis of the meteor trail. Our fundamental quantities are the diffusion coefficients $D_\parallel$ and $D_\perp$ parallel and perpendicular to $\mathbf{B}$, which are controlled only by fundamental ion and electron parameters and the value of B, and which have nothing to do with geometry. In Ref. [1], in contrast, other quite different, diffusion coefficients play the dominant role, for reasons which are not at all clear. The assumptions in Ref. [1] concerning quasineutrality are consistent (although not transparently so) with mainstream plasma physics, and with Eqs. (15) and (16) of the present paper. However, the justification for other crucial assumptions, for example, those spelled out at the beginning of Sec. 3 of Ref. [1], which underpin the whole subsequent analysis, is not at all clear. In any case it is not surprising that discrepancies become manifest at an early stage. Thus our expression (12) for the effective diffusion coefficient D_{eff} occurring in the decay constant for the backscattered signal, obtained independently of and prior to any explicit expressions for $D_\parallel$

and $D_\perp$, is quite different in form to the corresponding expression equation (28) of Ref. [1]. Given that this is the experimentally measured quantity, the implications for interpretation of meteor trails would seem to be potentially quite serious, and the present paper will hopefully serve as a stimulus for a reevaluation of the theory and assumptions contained in Refs. [1–3].

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